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THESIS

A MODEL FOR THE DISTRIBUTION OF RIFLE FIRE

by

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A Model for
the Distribution of Rifle Fire

by

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ABSTRACT

A simplified mathematical model for the distribution of fire of the individual infantry rifleman is developed. The development of this model considers some of the human factors aspects of the rifle-rifleman system as well as the ballistics and external effects which influence marksmanship and the distribution of fire. The degradation of hit probability due to reased range is incorporated into the model. The distributions generally implied in the concepts of point fire and area fire are examined. The significant variables related to hit probability and the distribution or fire are identified and approximate functional relationships are developed for these variables.

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I. INTRODUCTION

The salient functions of land combat may be described as those functions which involve command and control, intelligence, firepower, mobility, and sustainability. The extent to which each of these functions is accomplished by a combat unit determines, in part, the overall combat effectiveness of that unit. Each of these broad functional areas has been the subject of extensive research, field experimentation, and combat testing.

The subject of firepower has been a matter of prime concern for centuries. Increased firepower, or more accurately, greater fire effectiveness has been sought as a means of achieving a higher damage or casualty rate for any given target. Both small arms weapon design and basic infantry tactics have attempted to increase lethality and hit probability for a given packaged weight of rifle and basic load of ammunition.

Current usage broadly classifies small arms fire as either point fire or area fire. Existing United States Army doctrine classifies a target as either an area target (one that occupies a large area in width or depth) or as a point target (one that occupies a relatively small area). Small arms fire, by doctrine, is classified as either concentrated fire (fire directed at a point target) or distributed fire (fire delivered in width and depth to cover an area). [Ref. 1] Because of the prevalence of the terms point fire and area fire in military literature, they will be used in subsequent discussion in lieu of the more dogmatic terminology.

The questions which most frequently arise in terms of hit probability are: Under what conditions does area fire result in greater hit probability? Under what conditions does point fire result in greater hit probability? A more basic problem which directly influences the answers to these questions is the determination of the distribution of fire of the individual rifleman. And, finally, since combat targets do not appear at fixed or predetermined distances; what is the effect of range (distance from firer to target) on hit probability?

The distribution of fire of the individual rifleman and the effect of range on hit probability are the primary subject areas of this paper. However, the conditions for employing either point fire or area fire will be discussed since they are directly related to the distribution of fire of the individual rifleman.

II. DISTRIBUTION OF FIRE

A. FIRE EFFECTIVENESS AND HIT PROBABILITY

For small arms fire to be effective it is apparent that a significant percentage of the rounds fired must land in the immediate vicinity of the target. For an individual weapon which fires a non-fragmenting projectile, the round must hit the target to achieve its maximum effect or pass reasonably close to the target (a near miss) to achieve any partial suppressive effect. An analysis of some simple hit probability models provides useful insight for subsequent analysis of fire distribution.

1. One Dimension - Target Location Known

Define Total Miss Distance as the distance from the center of the target to the actual strike of the round. Total miss distance can be considered to be the sum of the miss distances due to several random variables of which some of the more significant are: aiming errors, target location errors, errors due to terminal ballistics, and errors due to the effects of wind across the bullet trajectory. Since the majority of point targets engaged by the rifleman in combat are less than 500 meters distant [Ref. 2], the terminal ballistics effects and wind effects may be considered as contributing a relatively small amount to the total miss distance.

Let X, a random variable, represent total miss distance. For mathematical convenience assume miss distance about a linear target, with center at $x = 0$, to be normally distributed with mean μ and variance σ^2.

The probability of a hit with a single shot, P_H, is:

$$P_H = P_H(\mu,\sigma) = \int_I f_X(x;\mu,\sigma)\,dx$$

where the target interval is I and the probability density function of miss distance is $f_X(x;\mu,\sigma)$.

For a target of width 2L:

$$P_H = \int_{-L}^{L} \frac{1}{\sigma\sqrt{2\pi}} \, \mathrm{EXP}\left[-\frac{(x-\mu)^2}{2(\sigma)^2}\right] dx$$

It can readily be shown (Appendix A) using Leibnitz's Rule that:

$$\frac{\partial P_H}{\partial \mu} < 0 \qquad \text{IF} \qquad |\mu| < 0$$

$$\frac{\partial P_H}{\partial \mu} = 0 \qquad \text{IF} \qquad \mu = 0$$

$$\frac{\partial P_H}{\partial \sigma} > 0 \qquad \text{IF} \qquad \mu > L$$

$$\frac{\partial P_H}{\partial \sigma} > 0 \qquad \text{IFF} \qquad \frac{\mu L}{\mathrm{Tanh}^{-1}} > \sigma^2, \ \mu > L$$

In other words, for a target whose location is known (and hence, target location error = 0), shifting the mean of the miss distance from the center of the target always decreases hit probability. If the mean miss distance is beyond the edge of the target, increasing dispersion (up to a certain limit) will increase hit probability. The effect of changes in mean miss distance and dispersion on hit probability can be shown more easily in graphic form. Figure 1 indicates hit probability for various values of μ and σ, where miss distance is normally distributed. Both μ and σ are expressed as functions of half-target width.

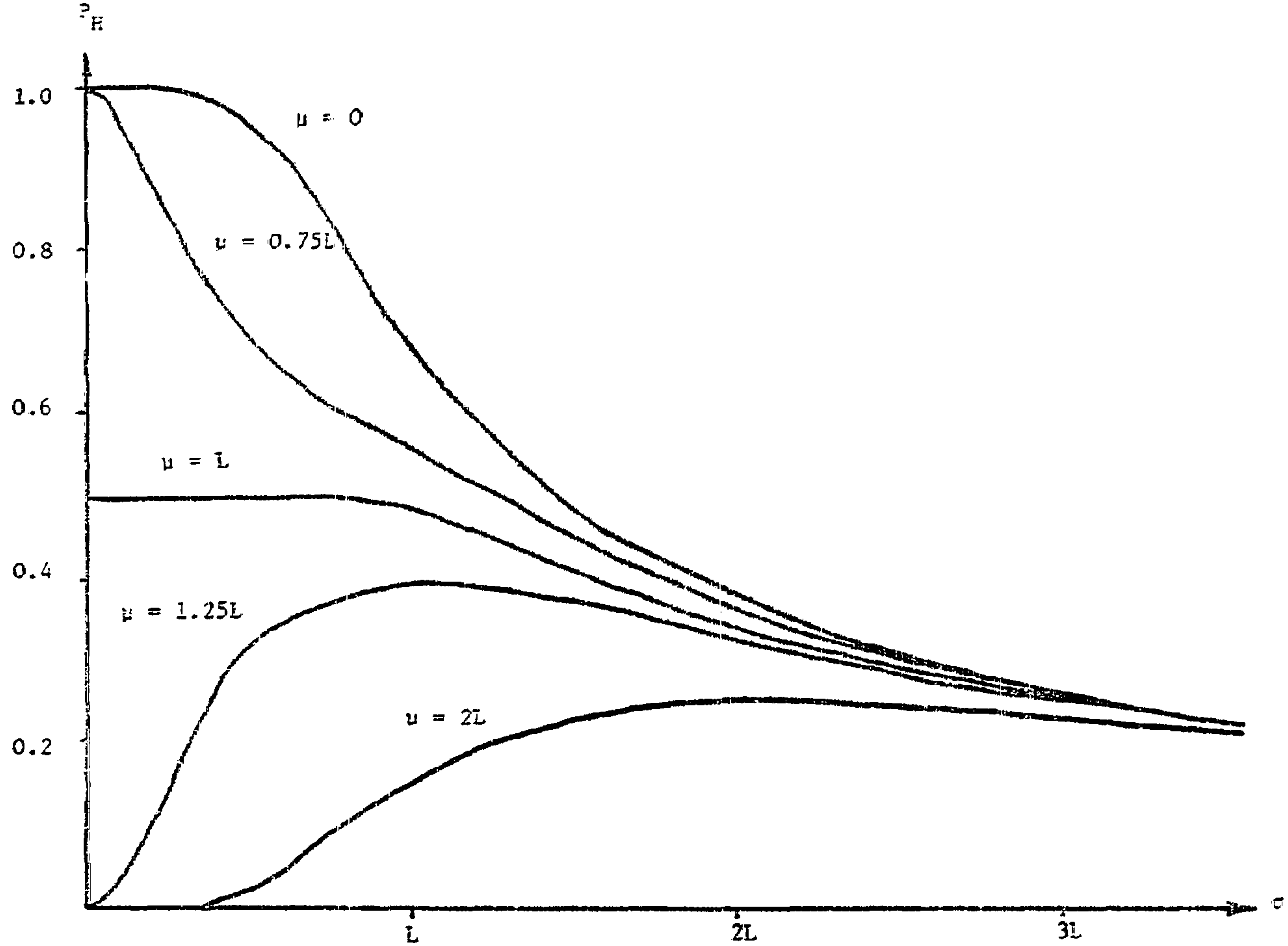

FIGURE 1

The practical significance of this analysis is that point fire should always be employed in preference to area fire when the location of the target is known; and, the aim point (which is approximately equal to the mean miss distance) should always be at the center of the target.

2. <u>One Dimension - Target Location Unknown</u>

Using the same definition for miss distance, consider a linear target of width 2L with center at Y, where Y is a random variable with some distribution in the interval (-a+L, a-L) and (-a, a) is the assigned sector of fire. If no information about the target exists, then any point in the interval can be considered equiprobable. Y may be considered as being uniformly distributed with probability density

$$f_Y(y) = \frac{1}{2(a-L)} \qquad -a+L<y<a-L$$

$$= 0 \qquad \text{otherwise}$$

The probability that Y is contained in some interval (c, d) where $-a+L<c<d<a-L$ is:

$$P[c<Y<d] = \int_c^d \frac{1}{2(a-L)}\,dy = \frac{d-c}{2(a-L)}$$

Recall that μ is approximately equivalent [illegible] the aim point when the target location error is zero. For μ and Y both located in some arbitrary interval (e, f), where $-a<e<$ [illegible] P_H will have a maximum value when $|Y-\mu| = 0$. For any given value of μ, P_H will have a minimum value when $|Y-\mu| = f-e$. As the size of the interval is made smaller, P_H can be made to approach its maximum value. The probability of Y being in the interval (e, f) depends only on the length of the interval not upon its location since Y is uniformly distributed.

Since the interval (e, f) was arbitrarily chosen, the range of values for P_H is the same no matter where the interval (e, f) is located within (-a, a). Thus, any particular range of values for P_H which includes the maximum value of P_H is the same for all locations for μ. This implies that μ should be uniformly distributed in the interval (-a+L, a-L).

The influence of dispersion on hit probability when the target location is unknown is more difficult to analyze. Using the concept of a region of uncertainty to reflect imperfect target acquisition, Schlenker and Olson [Ref. 3] discovered conditions for which the expected number of kills is the same for two significantly different values of σ where the values of σ were for two different type weapons, or equivalently, a weapon with two different type capabilities. The same method for any two values of σ can be used to investigate hit probability. Over a range of paired values of σ it would then be possible to quantify the influence of the magnitude of change in dispersion on hit probability when target location is unknown. An alternate procedure, outlined in Appendix B, is to formulate the problem of when to increase dispersion as a two-person game between the target and the firer.

A reasonable procedure for the rifleman, when the target location is unknown, is to select aim points uniformly within the area of suspected target location. In this sense the rifleman should use area fire. Should the location of the target become known, he should employ point fire.

3. Two Dimensions

Though more mathematically complex, a similar analysis can be made for two dimensional targets. It is r sonable to expect that the resulting conditions for employing area fire and point fire would be similar to those obtained for one dimensional cases.

B. DISTRIBUTION OF FIRE WITH RESPECT TO THE TARGET

The foregoing analysis made with respect to the target is reasonable given that the distribution of fire is normal. A similar procedure can be followed for the uniform distribution or any other non-pathological distribution. The obvious question is: What is the distribution of fire? Area fire seems to imply a uniform distribution in an area while point fire implies some concentration of rounds near the center of the target and, hence, some unimodal distribution. Thus, the uniform distribution and the normal distribution (with an appropriately selected value for σ^2) represent reasonable extremes for the selection of a probability distribution to mathematically model the distribution of fire. Alternatively, for large values of σ^2, a normal distribution, truncated at the left and right at the limits of the assigned sector, can also be made to model area fire within any specified degree of accuracy.

The inherent difficulty in this approach to model construction is that it attempts to determine the distribution of fire as if rounds were delivered by some random device. However, the random device is a rifleman who, through training and a habit pattern induced by constant training, behaves in a somewhat predictable manner. Another difficulty is that an analysis of fire distribution with respect to the target

yields a different distribution and a different hit probability model for each range. That hit probability decreases with increases in range is a well known fact.

These difficulties can be avoided if the semi-predictable nature of the behavior of the man-machine system, the rifleman, is used to make some logical inferences about the distribution of fire.

C. DISTRIBUTION OF FIRE WITH RESPECT TO THE FIRER

Consider the rifleman-rifle system. Let angular deflection from the rifle-center of target line to the rifle-point of impact line be a random variable. If the distribution of angular deflection and the functional relationship between angular deflection, range, and miss distance can be determined; then hit probability and the distribution of fire can be calculated for any given range.

1. The Training of a Rifleman

The sources of variability of round deflection may be considered to be caused only by the rifleman, the rifle, or external conditions. During an initial period of extensive training the rifleman is taught the importance of sight alignment, trigger control, breathing, and firing positions all to insure that the aim point at the time of fire is at the center of the target. The firing techniques learned during this initial training period and reinforced during all subsequent training periods, if used consistently, will minimize the variability of round deflection due to the rifleman. A marksmanship criterion frequently used during this training phase is the analysis of a three-round shot group (the pattern made by these rounds on the target) at 25 meters. From the size and shape of the shot group the types of

marksmanship techniques that are not being employed correctly can be determined. Shot group analysis is thus a valuable aid to training. The area that will completely encompass the shot group indicates the degree of proficiency attained in the application of the proper techniques of rifle marksmanship. At 25 meters a "satisfactory" (satisfactory in terms of being within minimum prescribed standards) shot group should fall within a circle of 3 cm diameter when fired from the prone supported or foxhole position and within a circle of 5 cm diameter when fired from the kneeling supported and all unsupported positions [Ref. 2].

During training "wobble" is defined as the movement of the rifle that occurs during aiming, and, "wobble area" as the extent of this movement in all directions [Ref. 2]. It is emphasized to the rifleman that this movement is a natural occurrence and cannot be completely eliminated. From the firer's viewpoint the wobble area is indicated by the movement of the front sight blade on and about the aiming point. The rifleman is taught that, if wobble becomes excessive causing the front sight blade to move completely off the target, he should hold trigger pressure until the front sight blade is on the target. Thus, even thought the actual movement of the front sight blade will describe a wobble area that is much larger than the area of the target; there exists a conscious effort on the part of the rifleman, as a result of thorough training, to fire only when the wobble area encloses the area of the target. This phenomena strongly suggests a normal distribution of angular deflection due to the rifleman. In two dimensions a bivariate normal or even a circular distribution is entirely reasonable.

The variability caused by the rifle (to include the ammunition) is chiefly due to terminal ballistics and has been successfully and accurately measured in tests of the rifle held in a mechanically locked position. The rifleman compensates for the variability of the rifle when he "zeros" his own rifle. After the pattern and size of the shot group have met minimum acceptable standards, the rifle sights are moved so that the center of the shot group is made to coincide with the center of the target.

The variability caused by external conditions is due primarily to wind effects although humidity, temperature, and light conditions will influence both the rifleman and the rifle. For the most part these latter external conditions have a negligible effect. Under 300 meters the effect of wind is also negligible unless the wind is of gale force [Ref. 2]. The rifleman can also compensate for wind effects by adjusting his rifle sights. Advanced training given primarily to snipers and competative shooters outlines techniques for calculating the effects of wind and the adjustments necessary to compensate for it. The ordinary rifleman, however, is taught to use an adjusted aiming point partly based on the location of the strike of the last round and partly on his own estimation of range and wind.

Thus, the firer, whether a sniper, competative shooter, or ordinary rifleman, employs some technique to compensate for variability due to both the rifle and external conditions. Because of this, the strong suggestion that angular deflection is normally distributed is unchanged.

2. Increasing Dispersion

Some comments seem appropriate concerning the ability of a rifleman to increase the amount of dispersion; since, as discussed previously, there are conditions when more dispersion increases hit probability. It is easily recognized that the rifleman, by selecting different aim points, can increase the dispersion of rounds in the target area. Whether he can increase the dispersion or variability of round deflection is a different matter. The dispersion or variability due to either external conditions or rifle ballistics cannot be altered by the firer. Thus he must, in effect, increase his own variability by either assuming a less stable position, changing his trigger squeeze, or adapting a less stringent aiming procedure.

In fact, there are combat situations when each of these measures is employed; but, only because the situation demands their employment such as: firing from the hip while moving forward during an assault or sighting over the top of the rifle at night to fully utilize night vision characteristics. To fire less steadily, with more jerking or flinching, or in any careless fashion as a means of inducing greater dispersion implies taking actions which are contrary to a strong habit pattern developed during training. Thus, to assume that a rifleman will purposely act in a manner to increase dispersion is not reasonable. Even at night, under conditions of extremely limited visibility, auxiliary aiming points are used to direct fire into likely enemy target areas. The muzzle flashes of enemy small arms also provide aiming points. Only if absolutely no knowledge exists and no fire is returned by the enemy should the rifleman fire in a completely random fashion.

The introduction of night sighting devices (infrared, etc.) practically makes such conditions nonexistent.

3. Initial Conclusions Concerning Fire Distribution

Some initial conclusions may be drawn at this point. Deflection may be considered to be normally distributed about the center of the target if its location is known. If the target location is not known, deflection may be considered to be normally distributed about the aim point.

Area fire (or, more accurately, distributed fire) is achieved by selecting aim points within the suspected target area such that the aim points are uniformly distributed within the target area. If the target location is known, point fire is preferred to area fire.

D. THE EFFECT OF RANGE

It is known that hit probability for small arms weapons decreases as range increases. Consider angular deflection, measured in radians, as being normally distributed about the center of the target. For convenience consider the special case of a circular distribution. Reference the line from the center of the target to the rifle at zero radians such that $\mu = 0$.

Now consider $Y = A \tan \alpha$, where α is measured in radians. For α small and $A = 1$

$$\tan \alpha \approx \alpha \qquad \text{or} \qquad Y \approx \alpha$$

and Y is approximately distributed normally with mean μ and variance σ^2.

In general, for small values of α:

$$Y \approx A\,\alpha$$

and $\qquad$ Y is approximately $N(A\mu,\ A^2\sigma^2)$

Let A represent the distance to the target. Then Y is approximately equal to miss distance.

If the MIL relation, which is based on the approximate equality of the length of the arc of a circle and the chord which subtends that arc, is used; then for small angles:

$$W \doteq R \times M$$

where W = miss distance in mm

R = distance to the target (range) in meters

M = deflection in mils

Then, given M is $N(\mu, \sigma^2)$:

$$W \text{ is approximately } N(R\mu,\ R^2\sigma^2)$$

and P_H can be calculated for any given range. The identical procedure can be followed for any bivariate normal distribution of deflection.

III. CONCLUSION

Many different types of mathematical models are currently being used to describe fire distribution. Some rely mainly on tabulated values of hit probability for various ranges obtained as a result of actual firings. Others rely more on stricter functional relationships between the many parameters. The degree of sophistication and complexity depends on the depth of the study being made.

The proposed model discussed in this paper is relatively free of mathematical complexity, can be used for all ranges, and takes into account not just the distribution of rounds on a target but includes some recognition of the human factors in the rifleman-rifle system. This model ignores the correlation between succesive rounds fired by a rifleman. (More precisely, it assumes complete independence.) It is felt that a positive correlation does exist; but the subject of round to round correlation is outside the realm of this paper. The proposed model has not been compared with field data to ascertain the acceptability of the hypothesized distribution of deflection. Prior to such a comparison it would be advisable to assume a bivariate normal distribution and to calculate hit probability based on the actual shape of the target. With these modifications a comparison can be made readily especially since data already exists that includes information about both hits near misses [Ref. 4].

An immediate application of this fire distribution model is its use in the simulation model originally developed for the United States Army Combat Developments Command Experimentation Command by Stanford Research

Institute and subsequently modified by the Litton Scientific Support Laboratory [Ref. 5]. This simulation model (LIVFIR) is used in conjunction with live fire ranges to assist in the quantification of fire effectiveness.

Finally, it is acknowledged that this model treats only the rifle and not any automatic weapons or machine guns. A similar analysis of the distribution of fire of these weapons would be desirable especially if this model for rifle fire is found to be practical.

APPENDIX A

PARTIAL DIFFERENTIATION OF P_H

As previously discussed, let X denote a random variable representing total miss distance. For X normally distributed about a linear target with center at x = 0:

$$P_H = \int_{-L}^{L} \frac{1}{\sigma\sqrt{2\pi}} \; e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $\frac{x-\mu}{\sigma} = t$, then $dt = \frac{dx}{\sigma}$. When x = L and -L, $t = \frac{L-\mu}{\sigma}$ and $\frac{-L-\mu}{\sigma}$ respectively. Then:

$$P_H = \int_{\frac{-L-\mu}{\sigma}}^{\frac{L-\mu}{\sigma}} \frac{1}{\sigma\sqrt{2\pi}} \; e^{-\frac{1}{2}t^2} \; \sigma \; dt = \frac{1}{\sqrt{2\pi}} \int_{\frac{-L-\mu}{\sigma}}^{\frac{L-\mu}{\sigma}} e^{-\frac{1}{2}t^2} \, dt$$

1. <u>With respect to μ</u>

$$\frac{\partial P_H}{\partial \mu} = \frac{1}{\sqrt{2\pi}} \left[e^{-\frac{1}{2}\left(\frac{L-\mu}{\sigma}\right)^2} \left(-\frac{1}{\sigma}\right) - e^{-\frac{1}{2}\left(\frac{-L-\mu}{\sigma}\right)^2} \left(-\frac{1}{\sigma}\right) + 0 \right]$$

Let $\lambda_1 = \frac{\mu}{\sigma}$ and $\lambda_2 = -\frac{L}{\sigma}$ Then:

$$\frac{\partial P_H}{\partial \mu} = \frac{1}{\sigma\sqrt{2\pi}} \left[-e^{-\frac{1}{2}(\lambda_1-\lambda_2)^2} + e^{-\frac{1}{2}(-\lambda_2-\lambda_1)^2} \right]$$

$$= \frac{e^{-\frac{1}{2}(\lambda_2+\lambda_1)^2}}{\sigma\sqrt{2\pi}} \left[1 - e^{2\lambda_2\lambda_1} \right]$$

$$\frac{e^{-\frac{1}{2}\left(\frac{\mu+L}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}} \left[1 - e^{\frac{2\mu L}{\sigma^2}} \right] < 0 \text{ since } e^x > 1 \text{ for } x > 0.$$

Therefore, P_H has a maximum at $\mu = 0$.

2. <u>With respect to σ</u>

$$\frac{\partial P_H}{\partial \sigma} = \frac{1}{\sqrt{2\pi}} \left[-e^{-\frac{1}{2}(\lambda_2-\lambda_1)^2} (\lambda_2-\lambda_1) + e^{-\frac{1}{2}(-\lambda_2-\lambda_1)^2} (-\lambda_2-\lambda_1) + 0 \right]$$

$$\frac{\partial P_H}{\partial \sigma} = \frac{e^{-\frac{1}{2}(\lambda_1-\lambda_2)^2}}{\sigma\sqrt{2\pi}} \left[(\lambda_1-\lambda_2) - (\lambda_1+\lambda_2)\ e^{-2\lambda_1\lambda_2}\right]$$

Since $\dfrac{e^{-\frac{1}{2}(\lambda_1-\lambda_2)^2}}{\sigma\sqrt{2\pi}}$, $(\lambda_1+\lambda_2)$, and $e^{-2\lambda_1\lambda_2} > 0$; a <u>necessary condition</u> for $\dfrac{\partial P_H}{\partial \sigma} > 0$ is: $(\lambda_1-\lambda_2) > 0$ which is equivalent to $\mu > L$.

A <u>sufficient condition</u> for $\dfrac{\partial P_H}{\partial \sigma} > 0$ is: $(\lambda_1-\lambda_2) > (\lambda_1+\lambda_2)e^{-2\lambda_1\lambda_2}$

or $e^{2\lambda_1\lambda_2} > \left(\dfrac{\lambda_1+\lambda_2}{\lambda_1-\lambda_2}\right) > 0$ since $\lambda_1 > \lambda_2$ (a necessary condition).

Thus: $$2\lambda_1\lambda_2 = \frac{2\mu L}{\sigma^2} > \ln\left(\frac{\lambda_1+\lambda_2}{\lambda_1-\lambda_2}\right) = \ln\left(\frac{\mu+L}{\mu-L}\right) > 0$$

or $$\frac{2\mu L}{\ln\left(\frac{\mu+L}{\mu-L}\right)} = \frac{\mu L}{\frac{1}{2}\ln\left(\dfrac{1+\frac{L}{\mu}}{1-\frac{L}{\mu}}\right)} = \frac{\mu L}{\operatorname{Tanh}^{-1}\left(\frac{L}{\mu}\right)} > \sigma^2$$ since $\operatorname{Tanh}^{-1} y = \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$

APPENDIX B

THE APPLICABILITY OF GAME THEORY TO DISPERSION ANALYSIS

The influence of dispersion on hit probability can be analyzed as a two-person constant-sum game in which Player I represents the firer and Player II represents the target. One possible game formulation would be based on the assumption that both players are rational and play according to the minimax principle. Consider such a game described as follows:

The firer (I) selects an aim point within some region R corresponding to a target area say, from -a to a. The target (II) selects a location, Y, in the same region. If miss distance from the center of the target, X, has some probability density function $f_X(x;\mu,\sigma)$; mean miss distance, μ, is approximately equal to the aim point; and σ is a fixed value; then, for a target of width 2L:

$$P_{H|M,Y} = \text{Prob [a hit} \mid M = \mu,\ Y = y] = \int_{y-L}^{y+L} f_X(x;\mu,\sigma)\, dx$$

Define the payoff function to I as: $V(\mu,y) = P_{H|M,Y}$. Then, since

$$\text{Prob [a miss} \mid M = \mu,\ Y = y] = 1 - P_{H|M,Y},$$

the payoff function to II is $1 - V(\mu,y)$ and the constant sum is 1. Thus the game is defined by the selection of the aim point, μ, by I; the selection of a location, y, by II; and the payoff functions, $V(\mu,y)$ and $1 - V(\mu,y)$.

Without any loss of meaning it is possible to transform all values for μ and y so that they are contained in the interval [0,1] and to adjust the values of L and σ accordingly. Label the payoff functions as a result of this transformation as $V^*(\mu,y)$ and $1 - V^*(\mu,y)$. Now the general procedure and theorems that apply to solving games on the unit square [Ref. 6 and 7] may be followed.

In general, I chooses μ from [0,1] by means of a distribution function, F; and II chooses y from [0,1] by means of a distribution function, G. The expectation of I for any given value of y will be:

$$\int_0^1 V^*(\mu,y)\ dF(\mu)$$

The total expectation of I, E(F,G) is:

$$E(F,G) = \int_0^1 \int_0^1 V^*(\mu,y)\ dF(\mu)\ dG(y)$$

The total expectation of II will be $1 - E(F,G)$ since the game is constant-sum. Player I will choose according to distribution F and the value of the game to him, v_1, is:

$$v_1 = \max_F \min_G E(F,G)$$

Similarly, Player II will choose according to distribution G and the value of the game to him, v_2, is:

$$v_2 = \min_G \max_F [1 - E(F,G)]$$

If the payoff function, $V^*(\mu,y)$ is continuous, it can be shown that $v_1 = v_2$ and it is possible to determine optimal mixed strategies

F^o and G^o for both Player I and Player II. (A more comprehensive treatment of continuous games may be found in Refs. 6 and 7).

Since the game just described was based on a fixed value for σ, it is possible to construct additional games, each with a different value for σ, and thereby analyze the influence of changes in dispersion on both hit probability and optimal aim point selection strategy (F^o). If a particular distribution is assumed for Player II, say G^*, then the problem is not a true game but more one of optimization and the value to Player I would be:

$$v_1 = \max_F E(F, G^*)$$

An entirely different class of games can be defined if Player I has exactly n rounds. In this case, since it is doubtful that Player II could move from one location to all other locations within the target area, some restriction placed on his sequence of choices would be appropriate. If Player II can return fire and has exactly m rounds, still another class of games is defined. Before game theory is used as an analysis technique the exact restrictions that define a particular class of games must be specified.

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13. ABSTRACT

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14. KEY WORDS	LINK A		LINK B		LINK C	
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